

# Bearing Similarity Measures for Self-Organizing Feature Maps

Narongdech Keeratipranon and Frederic Maire

Faculty of Information Technology  
Queensland University of Technology  
Box 2434, Brisbane Q 4001, Australia  
n.keeratipranon@student.qut.edu.au, f.maire@qut.edu.au

**Abstract.** The neural representation of space in rats has inspired many navigation systems for robots. In particular, *Self-Organizing (Feature) Maps* (SOM) are often used to give a sense of location to robots by mapping sensor information to a low-dimensional grid. For example, a robot equipped with a panoramic camera can build a 2D SOM from vectors of landmark bearings. If there are four landmarks in the robot's environment, then the 2D SOM is embedded in a 2D manifold lying in a 4D space. In general, the set of observable sensor vectors form a low-dimensional Riemannian manifold in a high-dimensional space. In a landmark bearing sensor space, the manifold can have a large curvature in some regions (when the robot is near a landmark for example), making the Euclidean distance a very poor approximation of the Riemannian metric. In this paper, we present and compare three methods for measuring the similarity between vectors of landmark bearings. We also discuss a method to equip SOM with a good approximation of the Riemannian metric. Although we illustrate the techniques with a landmark bearing problem, our approach is applicable to other types of data sets.

## 1 Introduction

The ability to navigate in an unknown environment is an essential requirement for autonomous mobile robots. Conventional *Simultaneous Localization and Mapping* (SLAM) involves fusing observations of landmarks with dead-reckoning information in order to track the location of the robot and build a map of the environment [1]. *Self Organizing (Feature) Maps* (SOM) are capable of representing a robot's environment. Sensor readings collected at different locations throughout the environment make up the training set of the SOM. After training, self-localization is based on the association of the neurons of the SOM with locations in the environment [2, 3]. Robustness to noise in the sensors can be achieved with probabilistic methods such as Extended Kalman Filters [4–7] or Particle Filters [8–10].

Navigation systems based on range sensors such as radar, GPS, laser or ultrasonic sensors are significantly more expensive than navigation systems relying only on vision [11–13]. An omni-directional vision sensor is composed of a digital

camera aiming at a catadioptric mirror. Although it is not straightforward to obtain distance estimations from omni-directional images due to the shape of the mirror, the bearings of landmarks relative to the robot are reasonably accurate and easy to derive from omni-directional images [14–16].

The Euclidean metric is the default distance used in most neural network toolboxes. Unfortunately, using this distance to train a SOM on a data-set of landmark bearing vectors collected at different locations uniformly distributed throughout the environment does not produce a grid of neurons whose associated positions in the environment are evenly distributed.

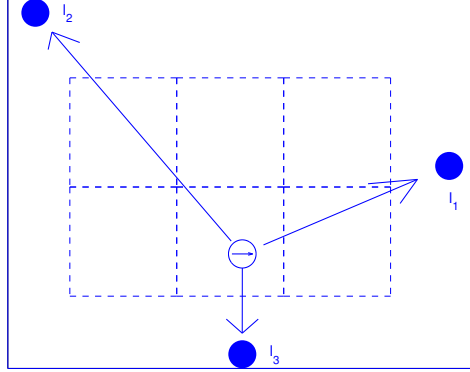
This is not very surprising as the Euclidian distance gives the same importance to all the components of the sensor vectors (here landmark bearings). It is intuitively clear that is not the right thing to do. The further a landmark is from the robot the larger the importance of its bearing becomes, as the bearing of a near landmark changes more wildly than the bearing of a far landmark when the robot is in motion. Therefore, we should give a relatively large weight to a far landmark and a relatively small weight to a near landmark. But, what values should these weights take? How to determine them in practice? In this paper, we provide answers to those questions.

Section 2 relates three landmark bearing vector metrics to probabilistic classifiers. In the same section, we present numerical experiments comparing these metrics. Section 3 outlines a method for estimating the intrinsic metric of a Riemannian manifold of sensor inputs. Section 4 concludes this paper.

## 2 Similarity measures for bearing vectors

In this section, we present three different methods to assess the similarity between two vectors of landmark bearings. The motivation of this project was to give a robot a sense of location and distance by building a SOM. The input vectors of the SOM are vectors of landmark bearings. Figure 1 illustrates the environment of the robot. In this example, the robot roves in a room equipped with 3 landmarks. The robot collects training sets by performing random walks. For a  $2 \times 3$  rectangular grid SOM, we could expect that the neurons of a trained SOM would be uniformly distributed. That is, the neurons should end up at the centres of the rectangular cells of Figure 1. Experiments with real and simulated robots show that the SOM fails to spread uniformly (with respect to the ground) if the Euclidian distance is used in sensor space.

To better understand the causes of the failure of the SOM to spread uniformly, we have investigated the shapes of the cells induced by bearing vector prototypes corresponding to regularly spaced observations on the ground. Observations were collected throughout the environment of Figure 2. The ground is partitioned into  $15 = 3 \times 5$  equal size grid cells. The average direction to each landmark in each cell are represented by the arrows at the centres of the cells. The length of an arrow represents the importance of the pointed landmark. The computation of this importance value is explained later in the paper.



**Fig. 1.** A toy model of a robot environment.

The  $15 = 3 \times 5$  mean vectors of landmark bearings in the different cells play the role of SOM neurons. The corresponding Voronoi diagram computed with the Euclidian distance on the bearing vectors is shown in Figure 3. We observe that with the Euclidian distance a large proportion of points get assigned to an incorrect cell centre. In this context, the localization problem can indeed be cast as a classification problem. Given a new bearing vector  $x$ , determining the cell in which the observation was made reduces to computing the probabilities  $P(i|x)$  that the observation has been made in the different rectangular cells  $i$  of Figure 2.

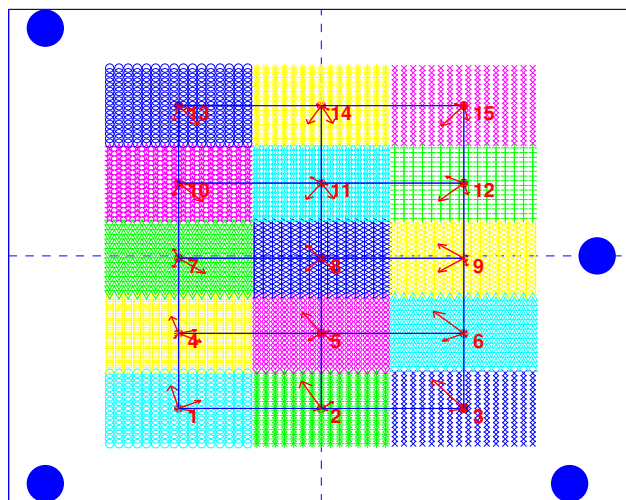
A Naive Bayes classifier provides a principled way to assign weights to the different landmarks. Recall that a Naive Bayes classifier simply makes the assumption that the different features  $x_1, \dots, x_4$  of the input vector are conditionally independent with respect to class  $i$ . That is,  $P(x_1, \dots, x_4|i) = P(x_1|i) \times \dots \times P(x_4|i)$ . With this class conditioned independence hypothesis, the most likely cell  $i$  is determined by computing  $\arg_i \max P(x_1|i) \times \dots \times P(x_4|i)$ . Let  $\mu_{ji}$  be the mean value of  $x_j$  observed in cell  $i$ , and let  $\sigma_{ji}$  be the standard deviation of  $x_j$  (the bearing of landmark  $j$ ) observed in cell  $i$ . Then, we have

$$P(x_j|i) = \frac{1}{\sqrt{2\pi}\sigma_{ji}} e^{-\frac{(x_j - \mu_{ji})^2}{2\sigma_{ji}^2}}$$

For the classification task, we compute the Naive Bayes pseudo-distance between the input vector  $x$  and  $\mu_i$  the bearing vector prototype of cell  $i$ ;

$$-\log\left(\prod_j P(x_j|i)\right) = \sum_j \frac{(x_j - \mu_{ji})^2}{2\sigma_{ji}^2} + \theta_i \quad (1)$$

where  $\theta_i = \sum_j \log(\sqrt{2\pi}\sigma_{ji})$ .



**Fig. 2.** An environment partitioned into  $15 = 3 \times 5$  equal size grid cells. The four large dots are the landmarks.

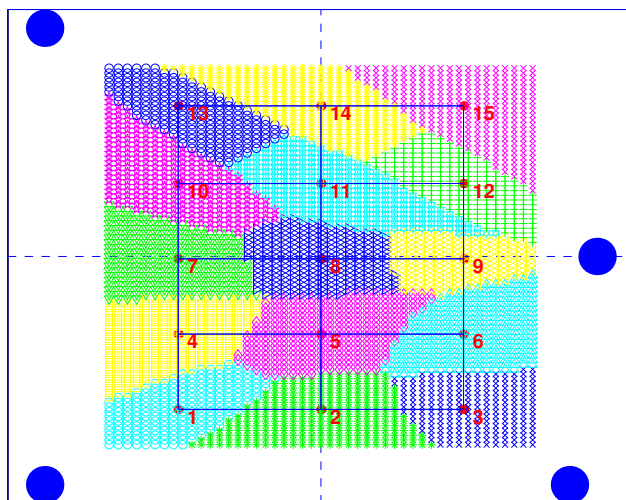
For this pseudo-distance, the weight of the bearing of a landmark  $j$  is determined by the standard deviation of the sample collected in the cell. The right hand side of Equation 1 is in agreement with our intuition that the further a landmark  $j$  is, the more significant the difference  $(x_j - \mu_{ji})^2$  becomes. Indeed the further the landmark is, the larger  $\frac{1}{2\sigma_{ji}^2}$  is.

Figure 4 shows the Voronoi diagram induced by Naive Bayes pseudo-distance on landmark bearing vectors is more in agreement with the Euclidian distance on the ground (compare Figures 3 and 4).

Further improvement in the accuracy of the localization can be obtained by estimating the covariance matrices of the sensor vector random variable in the different cells of Figure 2. Let  $C_i$  denote the covariance matrix of the  $n$ -dimensional bearing vectors collected in cell  $i$ . The general multivariate Gaussian density function for a sensor vector  $x$  observed in cell  $i$  is given by

$$P(x|i) = \frac{1}{(2\pi)^{\frac{n}{2}} |C_i|^{\frac{1}{2}}} e^{-\frac{1}{2}(x-\mu_i)^T C_i^{-1} (x-\mu_i)} \quad (2)$$

The Mahalanobis distance is obtained by taking the negative logarithm of Equation 2. The Mahalanobis distance has been successfully used for a wide range of pattern recognition and data mining problems. It has been also extended to mixed data [17]. In robotics, the Mahalanobis distance has proved useful for the data association problem [18, 19]. In previous work, the covariance matrix



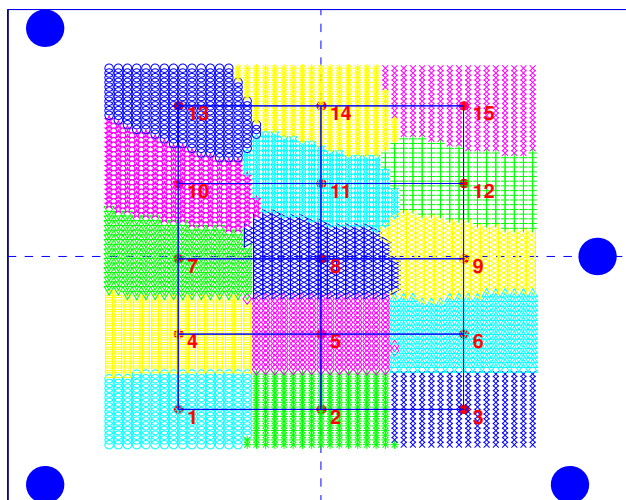
**Fig. 3.** Voronoi diagram computed with the Euclidian distance on the bearing vectors. The induced partition is quite different from the ideal partition of Figure 1. In particular, observation vectors corresponding to points close to the centre of cell 1 are incorrectly assigned to cell 2.

was used differently. It was used to model the noisy sensors. That is, given the Cartesian coordinates of the robot and the landmarks, measurements are repeated (without moving the robot) to estimate the noise in the sensors. Our approach is fundamentally different. The covariance matrix of the bearing vectors collected in a given cell  $i$  provides us with some information on the geometry of the manifold around this point in sensor space.

Figure 5 shows the classification results when using the Mahalanobis distance. Our experimental results (in simulation) show that the Euclidean distance achieves a classification accuracy of 92.17% percents, whereas the Naive Bayes distances achieves a classification accuracy of 97.14% percents. However, the best results are achieved with the Mahalanobis distance which reaches an accuracy of 99.32%.

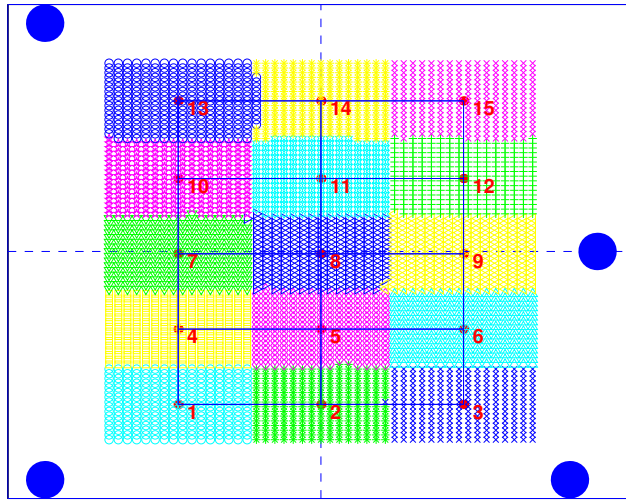
### 3 Strategy for approximating the Riemannian manifold

It is desirable to automatically build the partition of Figure 2. Unfortunately, a training set of observations does not allow us to directly compute the covariance matrices (assuming we have only access to bearing information). Moreover, we already saw that using a standard SOM training algorithm is not an option



**Fig. 4.** Voronoi diagram computed with the *Naive Bayes classifier* weighted distance on the bearing vectors. About 250 observations per cell were made for the evaluation of the statistical parameters of the Gaussian distributions.

(failure to spread the neurons evenly with respect to the ground). A possible strategy is to consider the weighted complete graph  $G$  whose vertices are the bearing vectors of a training set  $T$  and whose edge-weights are the Euclidian distance between the bearing vectors. Unfortunately, methods that build an auxiliary graph based on the  $k$ -nearest neighbors (like [20]) fail to build a proper grid for manifolds that have significantly different curvatures in orthogonal directions (the  $k$  nearest neighbors will be along the same direction). To address this problem, we build a graph  $G_\theta$  obtained from  $G$  by removing all edges whose weights are larger than  $\theta$ , (or equivalently setting the weights of those edges to  $\infty$ ). We then compute a grid-like subgraph  $H$  of  $G_\theta$  by imposing constraints on the relative positions of the neighbors. Preliminary experiments have shown that it is possible to compute  $H$  by simulating annealing using a fitness function based on the discrepancy of the degrees of the vertices of  $H$  (the desired degree is 4). A suitable representation of  $H$  for this search is as a union of a set of cycles of length 4 of  $G_\theta$ . From  $H$ , we can then estimate the distance between two bearing vectors on the manifold by computing the length of a shortest path in  $H$ . This work will be presented in a forthcoming paper.



**Fig. 5.** The best classification is achieved with the Mahalanobis distance.

## 4 Conclusion

In this paper, we have highlighted the differences between three natural similarity measures for bearing vectors. We have demonstrated the clear superiority of the Mahalanobis distance for localization based on bearings problems. We have also sketched a method for approximating the distance on the Riemannian manifold defined by a training set of sensor vectors. The approach presented in this paper is generic and not limited to localization from landmark bearing problems.

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