WHY DO STATISTICS EDUCATORS USE EXAMPLES TO TEACH STATISTICS?

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Examples have long been an integral component of statistics teachers' instructional repertoires but tend to be in the background of pedagogical knowledge. We explore the diverse ways that university statistics educators use examples, drawing on data from recent research (Gordon, Reid & Petocz, 2007). Three overlapping categories are proposed: examples are developed and presented by educators in basic instruction, examples are generated by students, under teacher direction, to aid learning and examples connect statistics with students' future professional work. Expressions in the second category were sparse suggesting an opportunity for statistics educators to develop teaching. We review models of exemplification in mathematics education and relate these to the empirical findings to begin the development of a framework for characterising examples in statistics education. We conclude that examples help promote statistical literacy.

1. INTRODUCTION

The use of examples to teach statistics at tertiary level is well documented in the published literature. These publications refer to various roles for examples. Niglas and Osula (2005) mention the importance of giving students examples of good statistical practice and examples to explain statistical concepts. Empirically they found that students valued practical examples to help them understand explanations, a finding that is echoed by Wulff and Wulff (2004). Online and journal resources for statistics education include data sets, applets and detailed examples that can be used in a lecture setting, such as resources published by causeweb.org (discussed by Bloomer Green, McDaniel & Holmes Rowell, 2005) and Teaching Bits in the Journal of Statistics Education. Macnaughton (2004) presents a conceptual approach to teaching introductory statistics emphasising relationships between variables. Primary concepts are presented to students in terms of numerous practical examples. Standard statistical principles and methods are then developed, again with emphasis on practical examples. Sowey (2001) and other educators demonstrate their ideas and provide exemplars as a resource for teaching statistics.

This brief 'sampling' of the statistics education literature suggests that the use of examples is central to the practice of teaching university statistics but is often in the background of pedagogical knowledge. Teachers may not necessarily articulate their reasons for using examples. Further, as Atkinson and Renkl (2007) point out, while example-based learning environments can be effective in supporting learning in some students, they may promote passive processing and hence poor student performance on subsequent, more complex tasks.

In this paper we present empirical data on why and how international university educators use examples based on recent research. We then review exemplification in mathematics and outline a framework relating these mathematical models to the empirical results. We discuss how examples help develop statistical literacy.

2. EXAMPLES IN EDUCATORS' REPORTS ABOUT TEACHING AND LEARNING STATISTICS AS A SERVICE COURSE

We report data from a research project investigating international educators' ideas about teaching statistics in 'service' courses—for students studying statistics as part of their various major study areas. Detailed descriptions of the methodology and project findings are available in publications on this project, including: Gordon, Reid & Petocz (2007); Gordon, Petocz Reid (2007), Petocz, Gordon & Reid (2006) and Reid, Petocz and Gordon (2008). Hence the methodology is summarised here only briefly.

Participation was invited through IASE (International Association for Statistics Education) and Faculty bulletin boards of Australian universities. In all, 36 IASE members from 12 different countries and 8 other Australian statistics educators took part in the project. The investigation consisted of a three-phase series of e-mail interviews based on an initial set of six open-ended questions. These questions included: *What do you consider to be the most important*

aspects of statistics for your teaching? and: *What are the attributes of a good statistics teacher at university?* Two rounds of follow-up questions probed participants' responses in depth enabling a personal dialogue in which educators could reflect on and expand their initial responses to questions and communicate their ideas.

Although participants were not specifically asked about their use of examples to teach statistics, two thirds of the educators spontaneously referred to examples in their e-interviews. From a content analysis of the interview transcripts we derived three categories for the use of examples from the respondents' perspectives: educators' examples used in basic instruction; student-generated examples as an aid to learning and examples presented to demonstrate statistics as a methodological tool in future professional work. There is no suggestion that these categories are mutually exclusive. Indeed, since the overall aim of teaching is to promote learning, all pedagogic tools are in essence learning tools. However, the categories are separated analytically to highlight the different foci that emerged from the data.

A. Examples as instructional devices used by educator

In this category the educator develops examples to engage students (and themselves), explain concepts or illustrate procedures, build skills and guide students' thinking. We summarise the varied uses of examples in this category in Table 1. All pseudonyms were chosen by participants themselves and brief excerpts are reported under these self-chosen pseudonyms.

Table 1: Ways that examples are used in instruction

Educators' use of examples	**Illustrative quote**
To engage students (and oneself)	Ron Fisher: *I am constantly updating my examples, and looking for new applications that will interest my students. Not only do I do this for the students' sake, it also makes the class much more interesting for me, since I am interested in the world around me.*
Illustrate ideas of lectures	Andrew: *The large methods courses have a one hour tutorial each week where examples are worked on that illustrate the lectures of the previous week.*
To ground concepts	Joyce: *[How do real world examples help students to learn statistics?] To use an educational psychology phrase, it gives them "an anchor".*
As a template for students to follow	Natalie: *By giving students worked examples they can use these as a "template" for their own work until they are comfortable creating their own non-technical explanations and conclusions.*
Develop skills	Margaret: *To develop good case examples for students to work on so that they develop their skills in a step by step fashion. To learn how to guide students and keep their interest as they go from simplistic examples to more complicated ones.*
Extra practice	Kay: *(Struggling) students get extra worksheets with examples.*
Differentiate statistics from mathematics	Primavera: *Students believe that Statistics is a branch of Mathematics. They change their mind with the use of real examples.*
Way into theory	Baz: *Even the students who can handle theory can learn from illustrative examples, and students who can't have no other choice.*
Develop critical thinking	Jane Johnson: *I sometimes use examples of incorrect analyses – as a warning to those who do not think critically.*
Build conceptual complexity	Despina: *I try to structure the problems I use as examples and as tutorial work so that we begin with a basic problem and slowly add complexity.* Kay: *We go over a number of examples, spread throughout the course – distributed versus massed practice, "spiralling" to repeat earlier concepts.*
Demonstrate the	John: *We will use an example reported in the media to illustrate how we*

statistical process	*can identify the statistical investigative process and understand statistical aspects of the study as reported.*
Indicate variation	Daria: *(We show students that variation is present in everything we do) by means of many "real" examples in the course of the lecture, and tutorials in the computer.*
Personalise teaching	Natalie: *Even though (many of us) use the same master resources, each of us adapt these materials slightly – add in our own examples, tweak to our own preferences.*

Many participants expressed the idea that the capability to develop and apply relevant and real examples—from the home discipline—is the hallmark of a good teacher. Conversely, poor examples, irrelevant to students' interests, can lose students. As Tilito summed up: *My students are from different disciplines, and all those examples of card or balls in probability, are obsolete, because they don't see where to apply them.*

B. Learner generated examples

Expressions fitting into this category were sparse.

Horace reported that if students could: *draw a picture, give the definition, state an example, and show they know when to use something, then that's a pretty good operational definition of understanding?! (If they can write the formula, that's a nice bonus!)*

Cara encouraged students: *to find examples of misuse (of statistics) on their own and present them in class.*

The few reports in this category indicated that student generation of examples helped students construct their own knowledge. As Janet Cole explained: *(The process) helps students put together/construct their own frameworks for learning. If a student can understand the process of constructing a confidence interval for a proportion (including checking conditions, mechanics, and presentation of results), then it should ideally be easier for that student to transfer this process to the construction of a confidence interval for any other situation. The student is not learning something entirely new, but is rather doing what I call a "variation on a theme."*

C. Examples presented to demonstrate applicability of statistics as a methodological tool in future professional work

In this category examples indicated the work of statisticians, or related to students' disciplines or future professional work.

Andrew: *It is important to convey to students the view that what they learn in a preparatory course on statistics is going to be essential for their future work in their major subject, something that many first year students do not appreciate. // Therefore, the statistics teacher must be able to access appropriate examples from the areas of application. //It is with these types of applications that students suddenly realise the greater picture of statistics. Such examples abound but they must be chosen very carefully. They must not be artificial class exercises in my view.*

Glee added that from his experience: *students respond very well to the statistics if the examples given in class relate to their specific disciplines. The motivation is that when they leave University they will be relevant and employable in their respective job markets.*

In turn, Johanna found that starting the semester by asking students why they think statistics has been made a compulsory unit in their degree program, led to discussion about the role of statistics in their particular field—how, when and why it would be used. *I use many examples that relate to the degree program that the students are enrolled in but also ask them to consider examples from different areas (highlights the diversity of statistical applications).*

John felt that in evaluating reports in the newspaper there should ideally be a balance of examples, some good reporting and some not such good ones. *Even although examples (of poor quality reports) demonstrate precisely what we want to make the students to be aware of, I don't feel comfortable about it—there is a certain element of glee and self-righteousness on our part when we come across such blunders in the newspaper.*

Statsboy: *So, in teaching statistics to medical students you MUST use medical examples – show me how the authors have analysed their data and what it means. Don't write a regression model with alphas and betas without giving me a relevant example of how this works in real life.*

Henry VIII: *What I try to show medical students is that, even if they don't ever intend to do any research, they still need some basic knowledge of stats in order to be able to fully understand the concepts of "statistical patterns" and "typical values", and the probabilistic nature of the decisions they have to make every moment during their practice. I try to do this by highlighting, through examples, the probabilistic nature of the patterns and decisions, and by trying to steer them away from the sort of deterministic thinking they are exposed to during most of the other courses they attend at college. // My most successful lecture on the Probability subject, for instance, is the one in which I discuss the interpretation of sensitivity, specificity and predictive value of diagnostic tests—these are very practical application of the rather abstract Bayes' theorem, and the students love it.*

The spontaneous reports about examples by participants, reported in section 2, indicate that the majority of them used examples, chosen by themselves, for a range of instructional goals. A few participants reported instances of examples generated by students in learning activities, while many educators used examples to indicate the utility of statistics in the student's own discipline.

The first two categories have much in common with the ways examples are used in mathematics and we review some models in the literature.

3. EXEMPLIFICATION IN MATHEMATICS

Bills et al (2006), in their overview of the role of examples in teaching and learning mathematics, give a broad definition of what constitutes an example—anything used as raw material for generalising, including intuiting relationships and inductive reasoning, illustrating concepts and principles, indicating a larger class, motivating, exposing variation and change and practising technique.

Examples play a central role in learning mathematics (Bills et al, 2006). Watson and Mason (2002a) note that people learn mathematics through engagement with examples rather than through formal definitions and techniques. They argue that it is only through this engagement that definitions assume any meaning. They advocate that students are exposed to a range of examples, as identifying the commonalities and differences in these examples is crucial for concept development (Watson & Mason, 2002b).

Bills et al (2006) argue that examples play a key role as communication tools between teacher and learner—fundamental to explanation and mathematical discourse—and their choice depends on factors including the teaching goals and the teacher's awareness of their learners' preconceptions and dispositions. However, the example chosen does not always fulfil its intended purpose. Mason and Pimm (1984) explored the inherent difficulties in teachers presenting their students with a generic example of a technique or theory. They argue that unless a teacher explicitly draws attention to the characteristics of the example that exemplifies the technique or theory, their students may focus on the particular example, resulting in students trying to "learn the example".

Recently, there has been a change in emphasis away from teacher-oriented to student-oriented activities in the mathematics classroom, leading to an increased emphasis on students constructing their own examples (Bills et al, 2006). Students may be asked to generate their own examples for assessment purposes or motivate interest in a new topic (Watson & Mason, 2002b). Watson and Mason (2002b) argue that student-generated examples can play an invaluable role in concept development. Dahlberg and Housman (1997) studied the use of student-generated examples in concept development in a mathematics undergraduate course. They showed that the generation of and reflection on examples provided powerful stimuli for learning. They found that students who employed an example generation learning strategy were more effective in attaining an initial understanding of a new concept than those who did not.

The ideas discussed above have implications for teaching statistics as well as highlight possible pitfalls about examples in learning statistics.

The central role of examples in mathematics has lead to a number of researchers characterising their use. Watson and Mason (2002b) proposed a framework of five types of intended experience for students when engaged with the task of constructing examples. Michener (1978) considered examples in a conceptual framework for understanding mathematics categorising (noteworthy) examples into four, not necessarily disjoint, epistemological classes: start-up examples that help motivate fundamental concepts and set up useful intuitions in a new subject; reference examples that are basic, widely applicable and used repeatedly in a branch of mathematics; model examples that are indicative of the general case and counterexamples which demonstrate what something is not. These classes resonate with some participants' reports of using examples to teach statistics, presented in section 2. However, a major difference is the emphasis on statistics as a tool for shedding light on real-life problems, rather than illustrating abstract mathematical concepts or theories.

Ideas about exemplification in mathematics education could develop example use in statistics pedagogy. We begin a framework, informed by ideas of Watson and Mason (2002b), Michener (1978) and the empirical categories that emerged from this study. The outline is sketched in Table 2. Illustrative examples fitting three boxes (a, b, c) follow the table.

Table 2: Framework for characterisation of examples in statistics education

Classification of example → Role of example in practice ↓	Start-up example for motivation	Developmental example to underpin/build concept or procedure	Example to illustrate generic structure	Counterexample
Educator's use in basic instruction		a		
Learner generated			b	
Methodological tool to demonstrate application in discipline	c			

Our data suggest some illustrations of the framework. John used examples from the media to underpin the "statistical investigative process" (box a); Janet Cole explained that student construction of a confidence interval for a proportion illustrates the generic structure of confidence intervals (box b); examples applying Bayes' theorem motivate the study of probability theory for medical students, according to Henry VIII (box c). The framework provides opportunities for statistics educators to reflect on, characterise and assess their own pedagogical use of examples.

4 CONCLUSION: EXAMPLES TO PROMOTE STATISTICAL LITERACY

Our empirical data show that statistics educators use examples in diverse ways for teaching including, importantly, promoting understanding of statistics applications in many disciplines. The findings suggest, too, that examples play an central role in enhancing statistical literacy, portrayed by Gal (2002) as the ability to interpret, critically evaluate, and communicate about statistical information and messages. Examples help develop understanding of concepts underpinning statistical knowledge, assist in developing awareness of context and promote critical thinking skills. Our participants' reports support many of these aspects, such as Kay's "spiralling" examples to develop concepts; Jane Johnson's "warning" examples of incorrect analyses and Statsboy's applications of regression models in a medical context. By engaging and

motivating students, examples enhance the affective components of learning statistics, which, Gal (2002) stresses, support the knowledge bases of statistical literacy. Respondents' expressions in Category C are at heart about making statistics personally meaningful to students.

Research is needed to assess the effects of different types of examples on student learning and how best to utilise examples for enhancing statistical literacy. Mathematics frameworks and models of exemplification could stimulate ideas on teaching statistics and expand statistics educators' repertoires in practice by offering a basis for a more reflective and theorised understanding of examples. Our framework begins this task.

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